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Strong Amplitude-Modulated Excitation of Resonance Oscillations in Duffing-type Oscillator

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ABSTRACT
A modified perturbation scheme is developed to investigate the effect of strong amplitude-modulated signal on the excitation of super- and sub-harmonic resonances in a Duffing-type system. The effect of varying amplitude of the primary and modulating signals on both super- and sub- harmonic frequency-response curves of the system have been investigated. The presence of modulating signal have been shown to induce newly generated super-harmonic resonance of order $r = 4, 5,$ and 6 in the system.

KEYWORDS
Duffing-Helmholtz Equation, Amplitude Modulation, Secondary Resonance.
2020 MSC: 34A34, ,70K25, 70K28, 70K40

1. Introduction

In recent years, Duffing-type oscillator has been extensively investigated as it is ultimately tends to simulate the behaviour of various physical systems [1, 2] and is used in various other engineering problems [3]. Resonance is one of the important dynamical phenomena exhibited by a forced nonlinear system . In the present work, we have considered the analysis of phenomenon of resonance in Duffing-Helmholtz system excited by an amplitude modulated signal. The analysis of primary resonance for harmonically excited Duffing-Helmholtz oscillator has been earlier studied extensively by many researchers in their respective fields [4–9]. Such an oscillator has also been studied earlier, in connection with the non-linear stellar pulsation [10, 11] and engineering problem [12].

In the present work, a given harmonically forced Helmholtz-Duffing system is first transformed into a simple Duffing-type system. Using a multiple time-scales perturbation method, we investigate the effect of strong forcing, *i.e.*, strong amplitude-modulated signal, on the newly generated secondary resonances in the transformed

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Duffing-type system. Subsequently, the frequency response curves for newly generated secondary super-harmonic resonances have been obtained and analyzed for different values of the amplitude of the amplitude-modulated forcing signal. In addition, the effect of the strong amplitude-modulated signal on the sub-harmonic resonance of order 3 has also been studied.

2. Response of the Duffing-type System to strong Amplitude Modulated Excitation(AME) using Modified Multiple-time Scale Perturbation Method (MMTPM)

A special type of nonlinear damped system, known as the Duffing-Helmholtz oscillator comprises of cubic as well as quadratic nonlinear terms [8, 9, 11, 13, 14]. Such a system is represented by

$$\ddot{Q} + 2\mu_0\dot{Q} + \omega_0^2 Q + I_1 Q^2 + I_2 Q^3 \quad = \quad 0, \tag{1}$$

where μ_0, ω_0 refer to the damping coefficient and the natural frequency, respectively whereas, I_1 and I_2 are the quadratic and cubic nonlinearity parameters of the system. In absence of the quadratic nonlinearity, the above written equation represents the Duffing Oscillator. The restoring force of Duffing-Helmholtz oscillator is characterized by asymmetric potential. To transform the asymmetric potential to a symmetric one, Cardano's transformation is applied to the Duffing-Helmholtz system and the resulting equation represents a Duffing-type system. We are interested in analyzing the secondary resonances in the system which is acted upon by an amplitude-modulated harmonic excitation, f. Applying the Cardano's transformation, $q(t) = Q(t) + \frac{I_1}{3I_2}$, the system (1) will now becomes harmonically excited Duffing system

$$\ddot{q} + 2\mu_0\dot{q} + \Omega_0^2 q + I_2 q^3 \quad = \quad f - k_0, \tag{2}$$

where $\Omega_0^2 = \omega_0^2 - \frac{I_1^2}{3I_2}$ represents the revised frequency, $k_0 = \frac{2I_1^3}{27I_2^2} - \omega_0^2\frac{I_1}{3I_2}$ may be considered as a steady bias for the oscillator and

$$f = [f_1 + 2g_1 \cos\Omega_M t]\sin\Omega t \tag{3}$$

is the time dependent AME supposed to be controlling the dynamics of the system externally.

In this work, we consider the following two settings:

(i) When the frequency of the modulation Ω_M is equal to the excitation frequency Ω, *i.e.*, $\Omega_M = \Omega$ and

(ii) when the frequency of the modulation Ω_M is not equal to the excitation frequency Ω and $\Omega_M << \Omega$.

As per the above mentioned situations, the AME f would be represented by

$$\text{(i) } f = f_1 \sin\Omega t + g_1 \sin 2\Omega t \quad \text{and} \quad \text{(ii) } f = (f_1 + 2g_1)\sin\Omega t, \tag{4}$$

respectively.

It may be noted that in situation (ii), the forcing functions appears as only harmonically excited frequency Ω but with a scaled amplitude *i.e.*, $f_1 + 2g_1$.

We now intend to take up the situation (i), when $\Omega_M = \Omega$ and MMTPM will be used for the analysis of AME induced secondary resonance in a strongly damped Duffing-type system. In this formalism, we invoke two transformations [15]. First one involves the transformation of the perturbation variable, ϵ, to a new variable, $\alpha = \alpha(\epsilon, a_0)$, as

$$\alpha = \frac{\epsilon a_0^2}{4(\Omega_0^2 + \frac{3}{4}\epsilon a_0^2)}, \qquad \text{such that} \qquad \epsilon = \frac{4\alpha}{a_0^2(1-3\alpha)}\Omega_0^2, \tag{5}$$

where a_0 is the amplitude of the fundamental harmonic. Therefore for large values of ϵa_0^2 , the value of $\alpha \to \frac{1}{3}$.

Second one involves Lindstédt-Poincaré transformation, $T = \Omega t$ [19,25,26] and following [11,17,27,28], square of the excitation frequency Ω^2 may be defined by the following expressions, as

$$\begin{aligned} \Omega^2 &= p\left(\Omega_0^2 + \frac{3}{4}\epsilon a_0^2\right)(1+\alpha\sigma), \\ \Longrightarrow \Omega^2 &= p\Omega_0^2\left(\frac{1}{1-3\alpha}\right)(1+\alpha\sigma), \end{aligned} \tag{6}$$

where p is a positive real number and σ is the detuning parameter [19]. Up to the first order Ω in terms of α may be written as

$$\Omega \simeq \sqrt{p}\Omega_0(1+\alpha\sigma_M) \qquad \text{with} \qquad \sigma_M = \frac{1}{2}\,(3+\sigma). \tag{7}$$

We call σ_M as the modified detuning parameter.

With the introduction of the new expansion parameter, α and Ω^2, defined in eqs.(5)-(6), respectively, we analyze the response of strongly nonlinear AME excited Duffing-type system which is now represented as follows:

$$\Omega^2 q'' + 2\epsilon\mu q' + \Omega_0^2 q + \epsilon I q^3 = (f_1 \sin T + g_1 \sin 2T) - \epsilon k. \tag{8}$$

Substituting for Ω^2, the above written equation becomes

$$\begin{aligned} (1+\alpha\sigma)q'' + 2\eta\alpha q' + \frac{1-3\alpha}{p}\,q + \frac{4\alpha I}{p}q^3 &= \frac{1-3\alpha}{p}(F_1 \sin T + G_1 \sin 2T) \\ &- \alpha\frac{K}{p}, \end{aligned} \tag{9}$$

where $'$ defines the derivative with respect to T. Further, we have used the definitions,

$$\eta = \frac{4\mu\Omega}{a_0^2 p}, \quad I = \frac{4I_2}{a_0^2}, \quad F_1 = \frac{f_1}{\Omega_0^2}, \quad G_1 = \frac{g_1}{\Omega_0^2}, \quad K = \frac{4k}{a_0^2}. \tag{10}$$

Next, the use of the multiple time-scale perturbation scheme, *i.e.*,

$$\begin{aligned} T(T_0, T_1, \cdots) &\equiv T(T_0, \alpha T_1, \cdots), \qquad \therefore \ \frac{d}{dT} = \frac{\partial}{\partial T_0} + \alpha \frac{\partial}{\partial T_1} + \cdots . \\ \frac{d}{dT} &= D_0 + \alpha D_1 + \cdots \ \text{and} \quad \frac{d^2}{dT^2} = D_0^2 + 2\alpha D_0 D_1 + \cdots \qquad (11) \\ q(T, \alpha) &= q_0 + \alpha q_1 + \cdots \end{aligned}$$

into eq.(9) results in the following set of equations,

$$\alpha^0 : \quad D_0^2 q_0 + \frac{1}{p} q_0 = \frac{(F_1 \sin T_0 + G_1 \sin 2T_0)}{p}, \qquad (12)$$

$$\begin{aligned} \alpha^1 : \quad D_0^2 q_1 + \frac{1}{p} q_1 &= -\sigma D_0^2 q_0 - 2D_1 D_0 q_0 - 2\eta D_0 q_0 + \frac{3}{p} q_0 \\ &\quad -\frac{I}{p} q_0^3 - 3\left(\frac{F_1 \sin T_0 + G_1 \sin 2T_0}{p}\right) - \frac{K}{p}. \qquad (13) \end{aligned}$$

Observe that eq.(12) could be viewed as a simple harmonic oscillator with natural frequency $1/\sqrt{p}$, excited by two superposed harmonic signals of amplitudes F_1/p and G_1/p with frequencies 1 and 2, respectively.
Solution for q_0(T) would now be

$$q_0(T) = A(T_1) e^{iT_0/\sqrt{p}} + \bar{A}(T_1) e^{-iT_0/\sqrt{p}} + X e^{iT_0} + \bar{X} e^{-iT_0} + Y e^{iT_0} + \bar{Y} e^{-iT_0}, \qquad (14)$$

where

$$A(T_1) = \frac{1}{2} a(T_1) e^{i\phi(T_1)}; \quad X = -ix, \quad Y = -iy \qquad (15)$$

with

$$x = \frac{F_1}{2(1-p)} \quad \text{and} \quad y = \frac{G_1}{2(1-4p)} \qquad (15(a))$$

$\bar{A}(T_1)$, $\bar{X}$ and $\bar{Y}$ are complex conjugates of $A(T_1)$, X and Y, respectively. Using eq.(14) into eq.(13) along with eqs.(15) results in the following expression, as

$$
\begin{aligned}
D_0^2 q_1 + \frac{1}{p} q_1 \;=\; & \left[\frac{a}{2p}(\sigma+3) - i\frac{a'}{\sqrt{p}} + \frac{a}{\sqrt{p}}\phi' - i\frac{a}{\sqrt{p}}\eta - 3\frac{I}{p}\left\{\frac{1}{8}a^2 + x^2 + y^2\right\} a\right] \\
\times\; & e^{i(T_0/\sqrt{p}+\phi)} \\
-\; & i\left[\sigma x + 2i\eta x + \frac{3}{p}x - 3\frac{I}{p}x\left\{x^2 + \frac{a^2}{2}e^{i\phi} + 2y^2\right\} + \frac{3F_1}{2p}\right] e^{iT_0} \\
-\; & i\left[4\sigma y + 4i\eta y + \frac{3}{p}y - 3\frac{I}{p}y\left\{y^2 + \frac{a^2}{2}e^{2i\phi} + 2x^2\right\} + \frac{3G_1}{2p}\right] e^{2iT_0} \\
-\; & i\,\frac{I}{p}\left[\frac{a^3}{8}e^{3\left(iT_0/\sqrt{p}+\phi\right)} + x\,\left\{x^2 - 3y^2\right\}e^{3iT_0} + 3x^2 y e^{4iT_0} + 3xy^2 e^{5iT_0}\right. \\
+\; & \left. y^3 e^{6iT_0}\right] - \frac{3I}{p}\left[axy\left\{e^{i\{(1/\sqrt{p}-1)T_0+\phi\}} + e^{i\{(1/\sqrt{p}+1)T_0+\phi\}}\right\}\right] \\
+\; & \frac{3I}{2p}\left[ax^2\left\{e^{i\{(1/\sqrt{p}-2)T_0+\phi\}} + e^{i\{(1/\sqrt{p}+2)T_0+\phi\}}\right\}\right] \\
+\; & \frac{3I}{2p}\left[ay^2\left\{e^{i\{(1/\sqrt{p}-4)T_0+\phi\}} + e^{i\{(1/\sqrt{p}+4)T_0+\phi\}}\right\}\right] \\
+\; & i\,\frac{3I}{4p}\left[a^2 x\left\{e^{i\{(2/\sqrt{p}-1)T_0+2\phi\}} - e^{i\{(2/\sqrt{p}+1)T_0+2\phi\}}\right\}\right] \\
+\; & i\,\frac{3I}{4p}\left[a^2 y\left\{e^{2i\{(1/\sqrt{p}-1)T_0+\phi\}} - e^{2i\{(1/\sqrt{p}+1)T_0+\phi\}}\right\}\right] \\
+\; & \frac{3I}{p}\left[axy\left\{e^{i\{(1/\sqrt{p}-3)T_0+\phi\}} + e^{i\{(1/\sqrt{p}+3)T_0+\phi\}}\right\}\right] + \text{c.c.} - \frac{K}{p}, \qquad (16)
\end{aligned}
$$

Here $'$ corresponds to the derivative of the argument with respect to time T_1. It may be observed that eq.(16) further provides the general secular term as

$$
\frac{a}{2p}(\sigma+3) - i\frac{a'}{\sqrt{p}} + \frac{a}{\sqrt{p}}\phi' - i\frac{a}{\sqrt{p}}\eta - \frac{3I}{p}a\left\{\frac{1}{8}a^2 + x^2 + y^2\right\} = 0. \qquad (17)
$$

Equation (17) suggests that the sub-harmonic resonance of order three, *i.e.*, $l = 3$ and the super-harmonic resonances of orders, $r = 3, 4, 5, 6$, are present for the AME excited Duffing-type system. Here, because of the presence of modulating signal, new super-harmonic resonance of order $r = 4, 5$ and 6 are observed in the frequency-response of the system.

2.1. *Super-harmonic Resonance*

In the case of third order super-harmonic resonance ($r = 3$), the secular term also includes the coefficient of $\exp(3iT_0)$ and hence, the complete secular term would be

$$
\frac{a}{p}\sigma_M - \frac{ia'}{\sqrt{p}} + \frac{a}{\sqrt{p}}\phi' - \frac{ia}{\sqrt{p}}\eta - \frac{3I}{p}\left\{\frac{1}{8}a^2 + x^2 + y^2\right\}\, a - \frac{iI}{p}\left\{x^3 - 3xy^2\right\}e^{i(\alpha\sigma_M - \phi)} = 0.
$$

Putting $\alpha\sigma_M - \phi = \gamma$, the secular term would be

$$a\left(\frac{1}{p}+\frac{1}{\sqrt{p}}\right)\sigma_M - i\frac{a'}{\sqrt{p}} + \frac{a}{\sqrt{p}}\gamma\,' - i\frac{a}{\sqrt{p}}\eta \;-\; \frac{3I}{p}a\left\{\frac{1}{8}a^2+x^2+y^2\right\} \\ -\; i\frac{I}{p}\left\{x^3-3xy^2\right\}e^{i\gamma} = 0. \qquad (18)$$

Now equating real and imaginary parts of eq.(18), we get

$$a\left(\frac{1}{p}+\frac{1}{\sqrt{p}}\right)\sigma_M - \frac{a}{\sqrt{p}}\gamma\,' - \frac{3I}{p}\left\{\frac{1}{8}a^2+x^2+y^2\right\}\,a + \frac{I}{p}\left\{x^3-3xy^2\right\}\sin\gamma = 0, \quad (19)$$

$$\frac{a}{\sqrt{p}}\eta + \frac{a'}{\sqrt{p}} + \frac{I}{p}\left\{x^3-3xy^2\right\}\cos\gamma = 0. \quad (20)$$

In steady state condition, last two equations, eqs.(19) and (20), reduce to

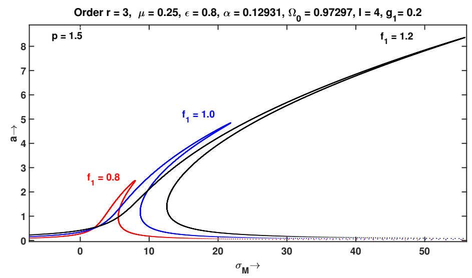

Figure 1. Frequency Response curves for $r = 3$ with $p = 1.5$ and $\Omega_M = \Omega$.

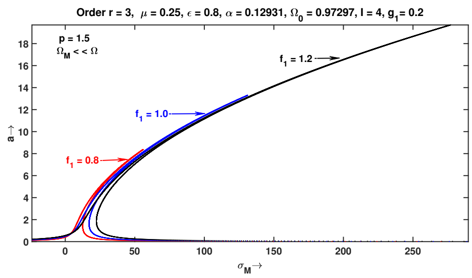

Figure 2. Frequency Response curves for $r = 3$ with $p = 1.5$ and $\Omega_M << \Omega$.

$$\frac{a}{p}\left(1+\sqrt{p}\right)\sigma_M - 3\frac{I}{p}\left\{\frac{1}{8}a^2+x^2+y^2\right\}\,a = -\frac{I}{p}\left\{x^3-3xy^2\right\}\sin\gamma, \quad (21)$$

$$\frac{a}{\sqrt{p}}\eta = -\frac{I}{p}\left\{x^3-3xy^2\right\}\cos\gamma. \quad (22)$$

The modified detuning parameter, σ_M, in the case of super-harmonic resonance of order $r = 3$, is given by

$$\sigma_M = \frac{1}{1+\sqrt{p}}\left[3I\left(\frac{1}{8}a^2+x^2+y^2\right) \pm \left\{\frac{I^2x^2}{a^2}\left(x^2-3y^2\right)^2 - \eta^2 p\right\}^{1/2}\right] \quad (23)$$

We have plotted the frequency response curves for both the situations, *i.e.*, $\Omega_M = \Omega$ and $\Omega_M << \Omega$ for comparison. Figures (1) and (2) are plotted for different values of f_1 with $p = 1.5$ which further clearly shows the distinction between the responses in the two cases sited earlier. Fig. 3 further illustrates the frequency response, for the case $\Omega_M = \Omega$, obtained for $p = 1.35$. To observe the effect of the amplitude of modulation, the frequency response curves are plotted for $g_1 = 0$ and 5 for fixed value of $f_1(=1)$ and $p(=1.5)$ (Fig. 4). It is found that the absence of g_1 reduces the amplitude, remarkably.
The increase in non-linearity parameter I_2 increases the amplitude and the resonant frequency, as well. Comparing the plots with different values of p, with $p = 1.5$ (Fig.5)

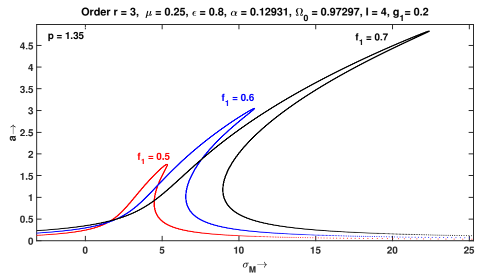

Figure 3. Frequency Response curves for $r = 3$ and different values of f_1.

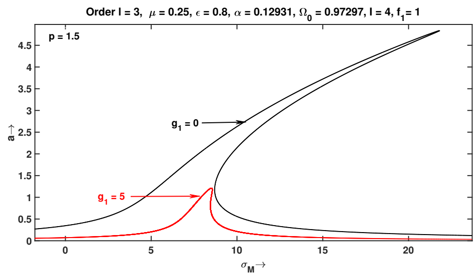

Figure 4. Frequency Response curves for $r = 3$ and different values of g_1.

and with $p = 1.35$ (Fig. 6), for varying I_2, it is observed that for larger value of p, the amplitude as well as, the resonant frequency are less, compared to smaller values of p. The coefficients of e^{4iT_0}, e^{5iT_0} and e^{6iT_0} contribute to the secular terms for the

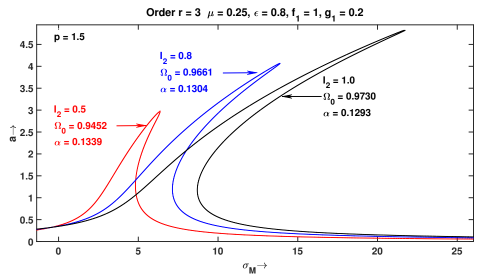

Figure 5. Frequency Response curves for $r = 3$ with $p = 1.35$ and different values of I_2.

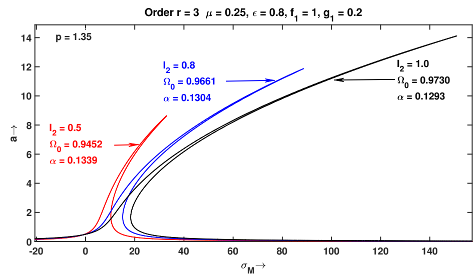

Figure 6. Frequency Response curves for $r = 3$ with $p = 1.5$ and different values of I_2.

newly generated 4th, 5th and 6th order super-harmonic resonances for $r = 4, 5$ and 6, respectively. Therefore, the modified detuning parameter σ_M for super-harmonic resonances in different cases could be written, respectively, as:

Super-harmonic Resonances of order $r = 4,\ 5$ and 6

$$\sigma_M(r=4) = \frac{1}{1+\sqrt{p}}\left[3I\left(\frac{1}{8}a^2 + x^2 + y^2\right) \pm \left\{\left(\frac{3I\ x^2 y}{a}\right)^2 - \eta^2 p\right\}^{\frac{1}{2}}\right], \tag{24}$$

$$\sigma_M(r=5) = \frac{1}{1+\sqrt{p}}\left[3I\left(\frac{1}{8}a^2 + x^2 + y^2\right) \pm \left\{\left(\frac{3I\ xy^2}{a}\right)^2 - \eta^2 p\right\}^{\frac{1}{2}}\right], \tag{25}$$

$$\sigma_M(r=6) = \frac{1}{1+\sqrt{p}}\left[3I\left(\frac{1}{8}a^2 + x^2 + y^2\right) \pm \left\{\left(\frac{I\ y^3}{a}\right)^2 - \eta^2 p\right\}^{\frac{1}{2}}\right]. \tag{26}$$

The plotted frequency response curves for super-harmonic resonance of order $r = 4$ also suggests an increase in the amplitude and the resonant frequency with increase in the amplitude f_1 of the excitation (figs. 7 & 8), as in the case of $r = 3$.

The frequency response curves for $r = 5$ and 6 (Figs. 9 & 10), show that an increase in excitation amplitude produces an increase in resulting amplitude and the resonant frequency, but, jump-up and jump-down phenomena are not observed for the given parameters.

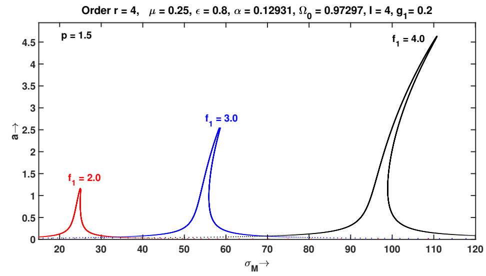

Figure 7. Frequency Response curves for $r = 4$ with $p = 1.5$.

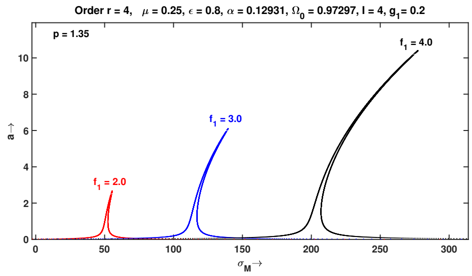

Figure 8. Frequency Response curves for $r = 4$ with $p = 1.35$.

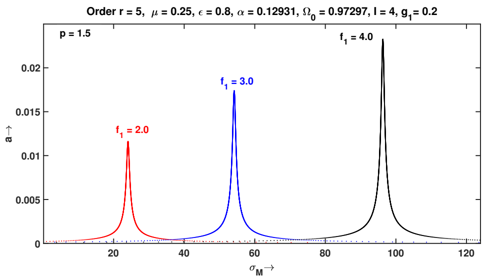

Figure 9. Frequency Response curves for $r = 5$.

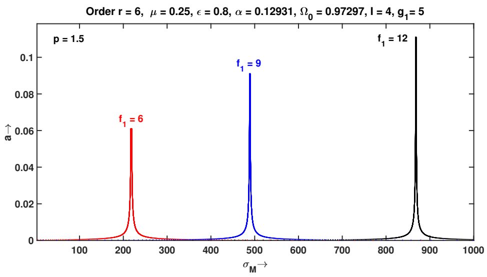

Figure 10. Frequency Response curves for $r = 6$.

2.2. *Sub-harmonic Resonances*

For the system we have considered, only a single sub-harmonic resonant state, *i.e.*, of order $l = 3$, is present. For $l = 3$, the secular term generated will include the coefficient of $e^{\{-i(2/\sqrt{p}-1)\mathrm{T}_0\}}$, which is $-\frac{3I}{p}\bar{A}^2X$. Therefore, now the secular term in this case would be

$$\frac{a}{p}\sigma_M - \frac{ia'}{\sqrt{p}} - \frac{ia}{\sqrt{p}}\eta + \frac{a}{3\sqrt{p}}\sigma_M - \frac{a}{3\sqrt{p}}\gamma\,' - \frac{3Ia}{p}\left\{\frac{1}{8}a^2 + x^2 + y^2\right\} - i\frac{3I}{4p}a^2xe^{i\gamma} = 0.$$

where $\gamma = \sigma_M T_1 - 3\phi$.

Equating real and imaginary parts from the above equation to zero, we obtain

$$\frac{1}{3\sqrt{p}}a\gamma\,' = \frac{3+\sqrt{p}}{3p}a\sigma_M - \frac{3I}{p}\left\{\frac{1}{8}a^2 + x^2 + y^2\right\}\ a + \frac{3I}{4p}a^2x\sin\gamma, \tag{27}$$

$$-\frac{a'}{\sqrt{p}} = \frac{a}{\sqrt{p}}\eta + \frac{3I}{4p}a^2x\cos\gamma. \tag{28}$$

In steady state conditions, eqs.(27) and (28) reduce to

$$\frac{3+\sqrt{p}}{3p}a\sigma_M - 3I\left\{\frac{1}{8}a^2 + x^2 + y^2\right\}\ a = -\frac{3I}{4p}a^2x\sin\gamma, \tag{29}$$

$$\frac{a}{\sqrt{p}}\eta = -\frac{3I}{4p}a^2x\cos\gamma. \tag{30}$$

and ultimately, we arrive at the expression for σ_M for sub-harmonic resonance of order

$l = 3$, as

$$\sigma_M = \frac{3}{3+\sqrt{p}} \left[3I \left\{ \frac{1}{8} a^2 + x^2 + y^2 \right\} \pm \left\{ \left(\frac{3I}{4} a x \right)^2 - \eta^2 p \right\}^{\frac{1}{2}} \right]. \tag{31}$$

The frequency response curves for sub-harmonic resonance of order $l = 3$ for different values of excitation amplitude f_1 and nonlinearity parameter I_2 are plotted for two settings, *i.e.*, $\Omega_M = \Omega$ and $\Omega_M << \Omega$. It is observed that the critical frequency gets shifted towards higher values for $\Omega_M << \Omega$ compared to the situation when $\Omega_M = \Omega$ (Fig. 11 - 14).

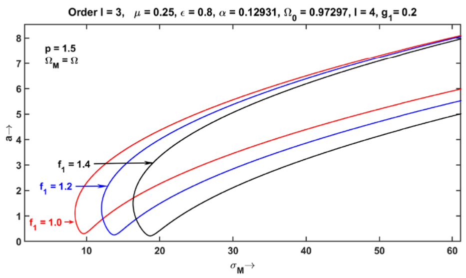

Figure 11. Frequency Response curves for $l = 3$ with $\Omega_M = \Omega$.

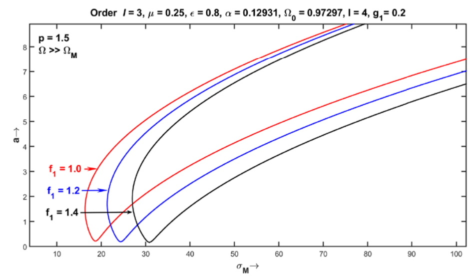

Figure 12. Frequency Response curves for $l = 3$ with $\Omega_M << \Omega$.

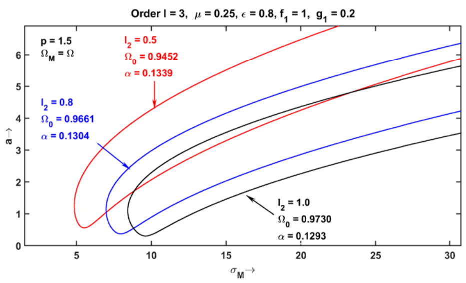

Figure 13. Frequency Response curves for $l = 3$ with $\Omega_M = \Omega$.

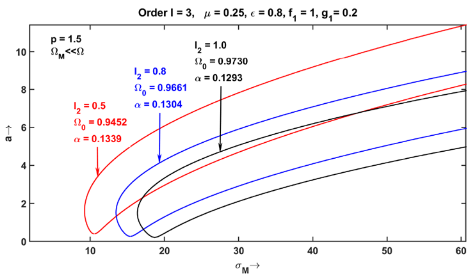

Figure 14. Frequency Response curves for $l = 3$ with $\Omega_M << \Omega$.

3. Concluding remarks

In this work, we have developed a frame-work to study the effect of harmonically excited nonlinear oscillator viz., Duffing-type oscillator on the secondary resonances of the system when the excitation amplitudes are very large. The framework involves mainly two steps. First one redefining the new expansion parameter, α, that takes care of very large amplitude of the excitation and second one involves the Lindestedt-Poincare transformation involving the primary frequency of excitation. Along with the multi-time scale perturbation, we find the analysis to be able to handle the setting where perturbation are strong enough to excite newly generated secondary resonances viz., super-harmonic resonances of order $r = 4, 5,$ and 6 in the system. Limiting ourselves to first order perturbation in terms of the new expansion parameter, α, we obtained analytical expression for the shift in super-harmonic resonance frequency of order $r = 3, 4, 5$, and 6. The frequency -response curves exhibiting hysteresis effect for various parameters are shown in Fig. 1-6. Fig.9 & 10 further illustrate the absence of the hysteresis effect for the specific parameters chosen. In case of sub-harmonic resonance of order $l = 3$, the frequency response curves (Fig.11 & 12) indicate a shift in the critical frequency as the parameter, f_1, values get increased for setting (i) and (ii). Further, the frequency response curves obtained in Fig.13-14 shows that for given values of f_1, g_1, the critical frequency also exhibits a shift with increase in the nonlinearity parameter, I_2, of the Duffing-type system. In a subsequent report , we plan to extend the work to higher order in the expansion parameter.

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